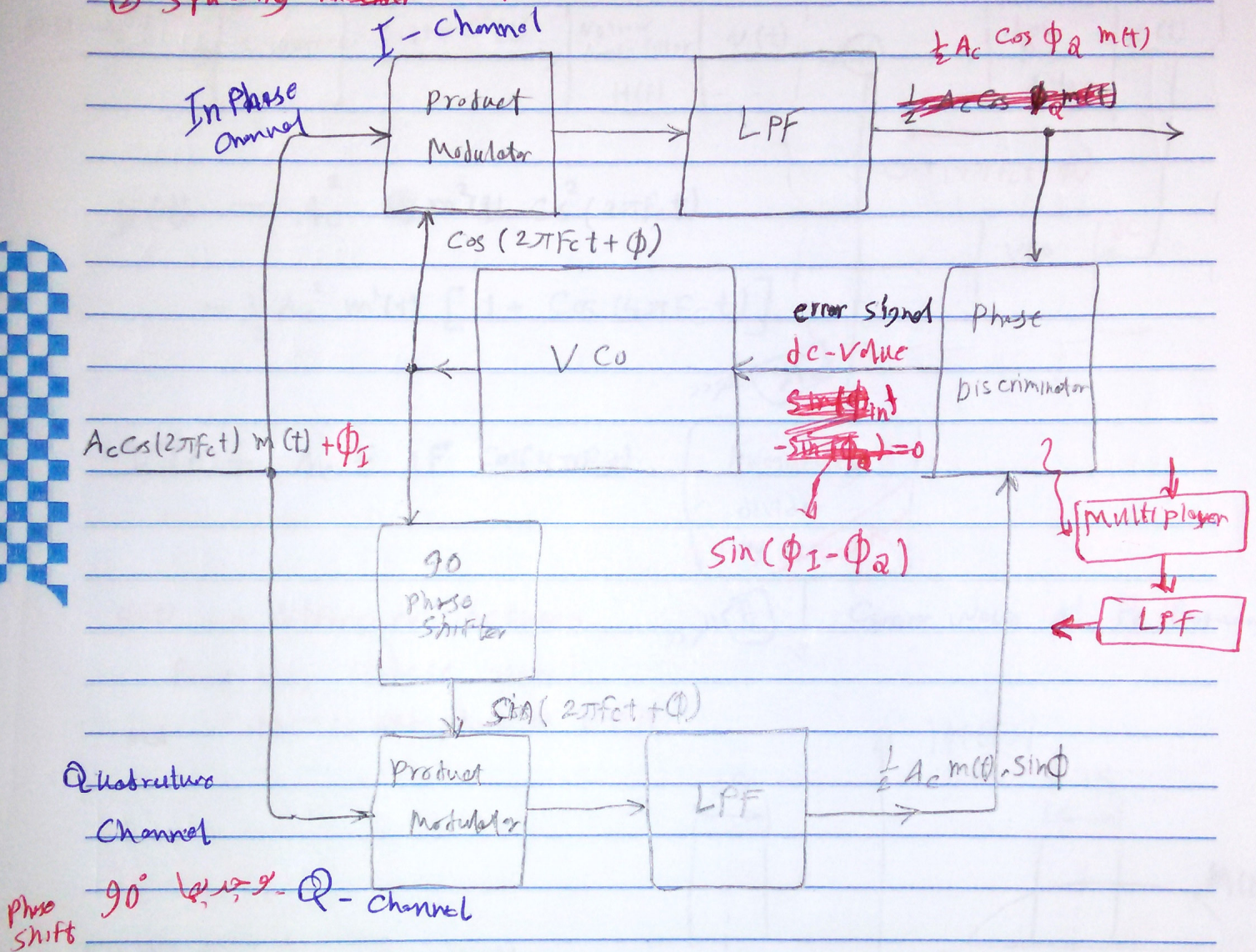


Phase-Shift Feedback

① Costas Receiver

② Squaring ~~Phase~~ Loop



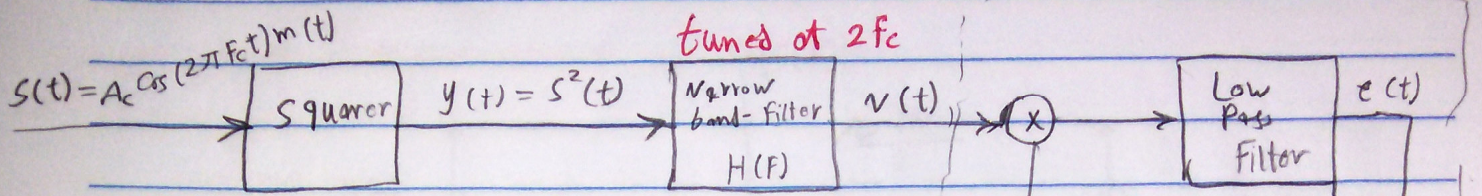
VCO \rightarrow Voltage Control Oscillator

phase-tracking

is a self

② Squaring Loop

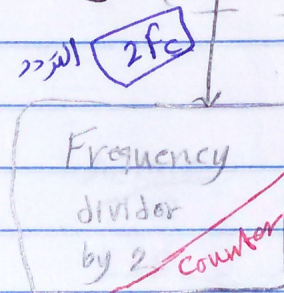
PLL



$$y(t) = A_c^2 m^2(t) \cos^2(2\pi F_c t)$$

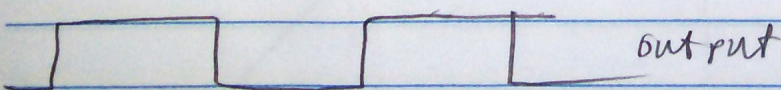
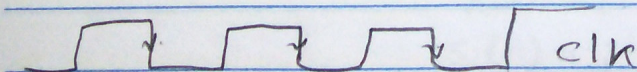
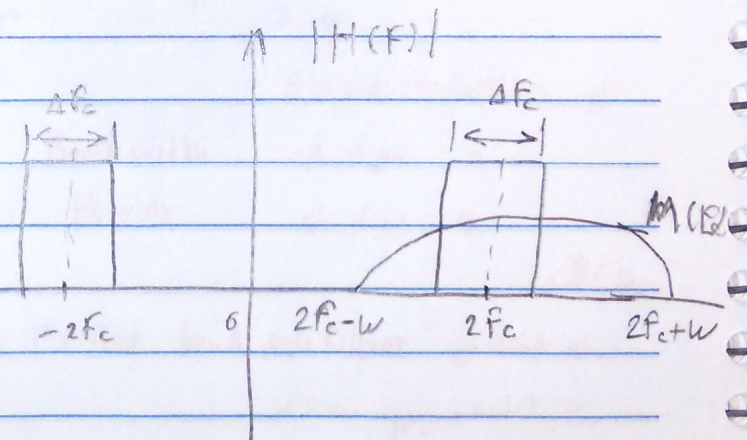
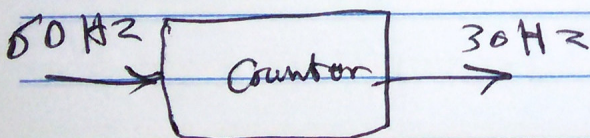
$$= \frac{1}{2} A_c^2 m^2(t) [1 + \cos(4\pi F_c t)]$$

$$V(t) = A_c E \Delta F \cos(4\pi F_c t)$$



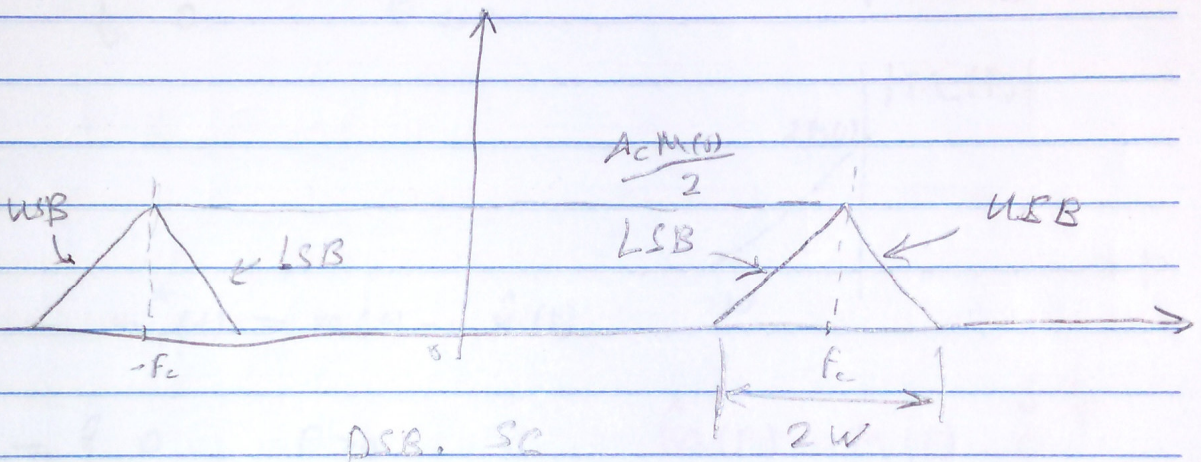
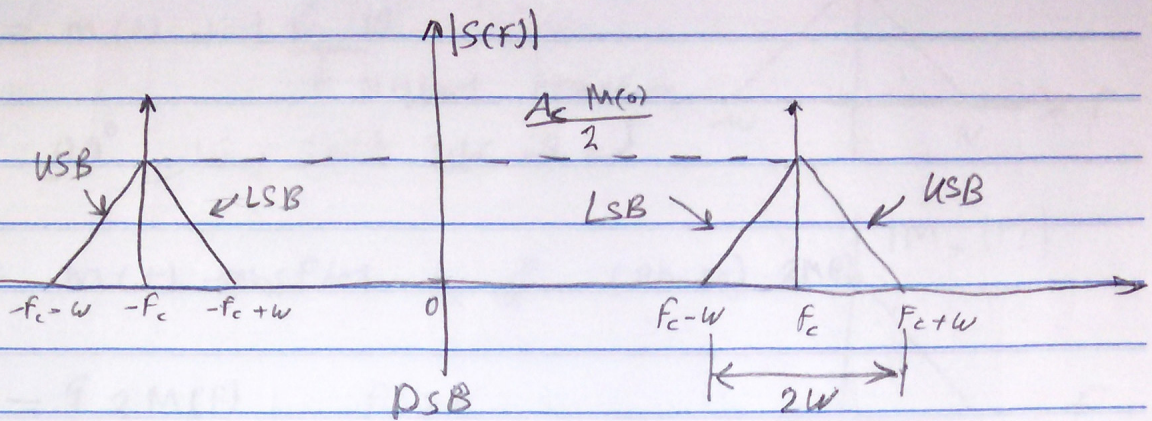
$e(t)$ → difference between
Frequency (phase $v(t)$)
and Frequency and phase of VCO

Carrier wave at F_c frequency



bit $\times 0$

Single side band Modulation (SSB)



في حال Single side band

* يوتر في Band width

* يوتر في Power

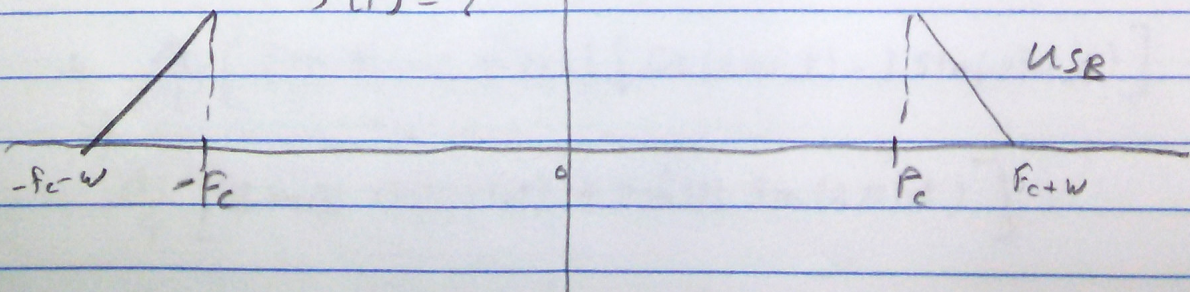
في حال Single side band

* يوتر في Band width

* يوتر في Power

$$S(t) = ?$$

$$S(f) = ?$$

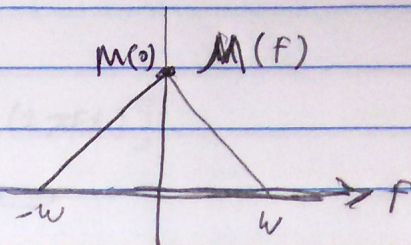


pre-envelope of $m(t)$:

$$m_+(t) = m(t) + j \hat{m}(t)$$

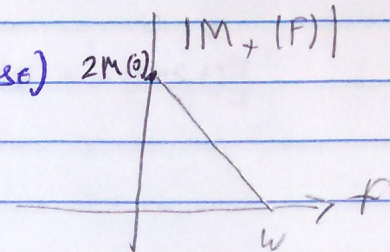
↓ Hilbert transform

90° phase shift (use of ϵ)



$$\hat{m}(t) = m(t) \text{ shifted by } \frac{\pi}{2} \text{ (Phase)}$$

$$M_+(f) = \begin{cases} 2M(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

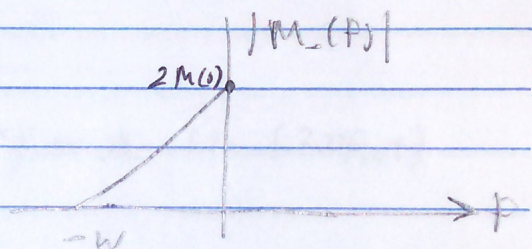


~~$M_-(f)$~~

$$m_-(t) = m_+^*(t) = m(t) - j \hat{m}(t)$$

$$M_-(f) = \begin{cases} 0 & f > 0 \\ 2M(f) & f < 0 \end{cases}$$

$$\hat{M}(f) = M(f) \cdot e^{j \frac{\pi}{2}}$$



To get single side band:

Upper Side Band

$$s(t) = \left[m_+(t) e^{j 2\pi f_c t} + m_-(t) e^{-j 2\pi f_c t} \right] \frac{A_c}{4}$$

$$s(t) = \frac{A_c}{4} \left[(m(t) + j \hat{m}(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t)) \right]$$

$$+ \frac{A_c}{4} \left[(m(t) - j \hat{m}(t)) (\cos(2\pi f_c t) - j \sin(2\pi f_c t)) \right]$$

$$= \frac{A_c}{4} \left[2m(t) \cos(2\pi f_c t) - 2\hat{m}(t) \sin(2\pi f_c t) \right]$$

$$= \frac{A_c}{2} \left[m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t) \right]$$

Lower Side Band

$$S(t) = \frac{A_c}{2} [m(t) \overset{\cos(2\pi f_c t)}{\cancel{\cos(2\pi f_c t)}} + \hat{m}(t) \sin(2\pi f_c t)]$$

For single side band

$$S(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t)]$$

Ex: $m(t) = A_m \cos(2\pi f_m t)$

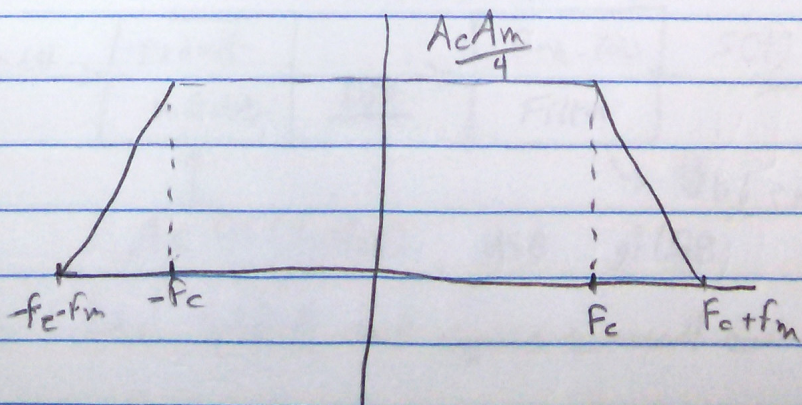
Amplitude A_m ~~and~~

$$\hat{m}(t) = A_m \cos(2\pi f_m t + \frac{\pi}{2}) = A_m \sin(2\pi f_m t)$$

$$S(t) = \frac{A_c}{2} [A_m \cos(2\pi f_m t) \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \sin(2\pi f_c t)]$$

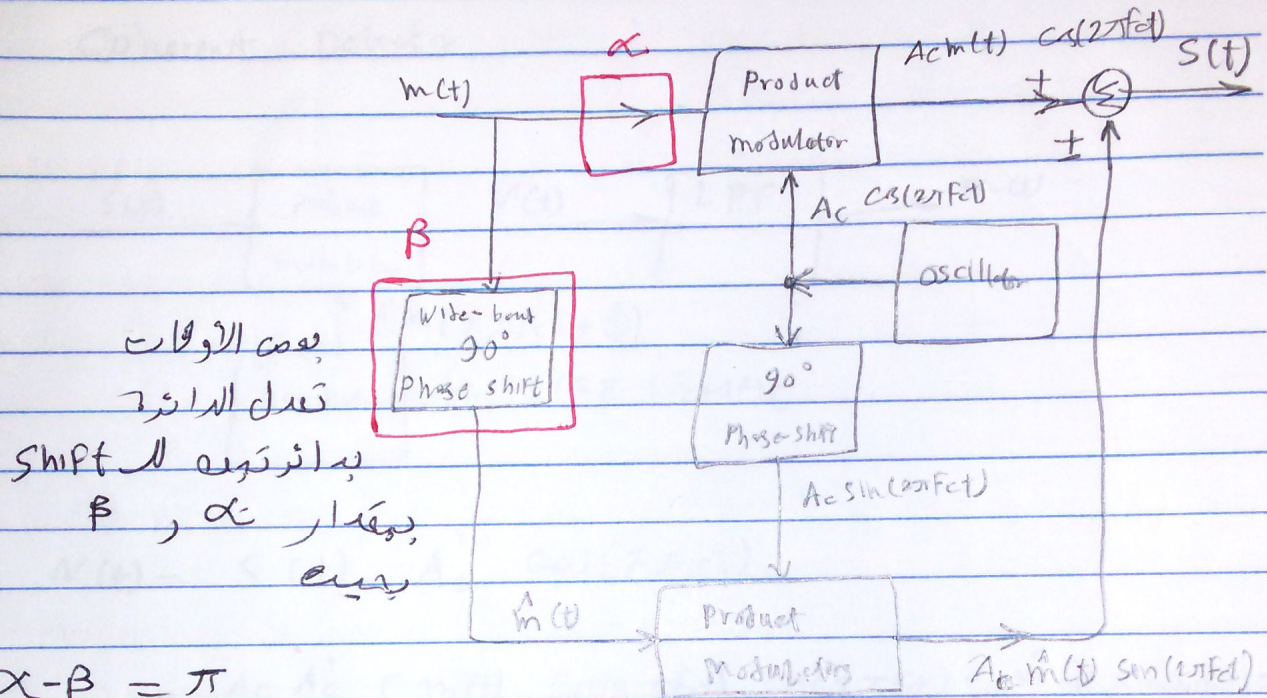
$$= \frac{A_c A_m}{2} \cos(2\pi (f_m + f_c) t)$$

$$S(f) = \frac{A_c A_m}{4} [\delta(f + f_c + f_m) + \delta(f - f_c - f_m)]$$



Generation of SSB Waves :

- ① phase discriminator
- ② Frequency discriminator



$$\alpha - \beta = \frac{\pi}{2}$$

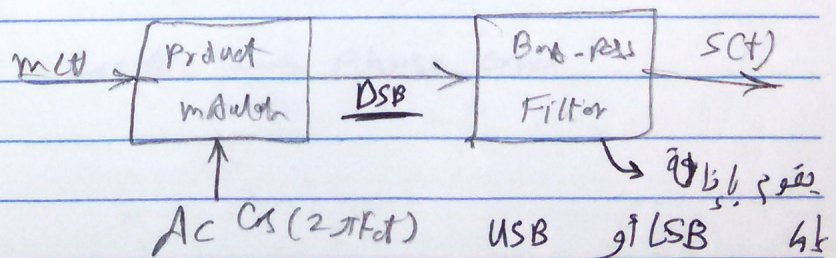
هذا يعني أن

استقبال ترددات

ولا يوجد أي إشارات أخرى

Phase discriminator

Frequency discriminator

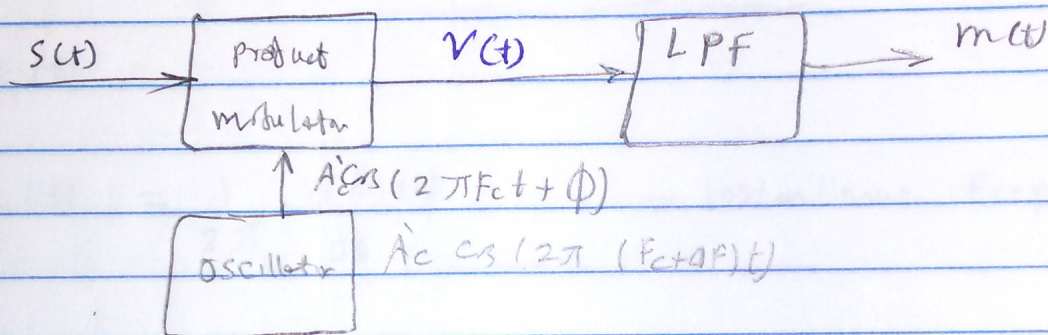


من الممكن أن يكون

The modulation of SSB :

$$S(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t)]$$

Coherent Detector



$$V(t) = S(t) \cdot A_c' \cos(2\pi f_c t)$$

$$= \frac{A_c A_c'}{2} [m(t) \underbrace{\cos(2\pi f_c t) \cos(2\pi f_c t)}_{\cos^2(2\pi f_c t)} \mp \underbrace{\hat{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t)}_{\frac{1}{2} \hat{m}(t) \sin(4\pi f_c t)}]$$

$$= \frac{A_c A_c'}{2} \left[\frac{1}{2} m(t) [1 - \cos(4\pi f_c t)] \right]$$

$$= \frac{A_c A_c'}{4} m(t) \cos \phi \rightarrow \text{Phase error}$$

Angle Modulation

$$S(t) = A_c \cos(\theta_i(t))$$

Am wave

$$S(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t + \phi)$$

$\phi \rightarrow 0$

$$\theta_i(t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \text{instantaneous frequency}$$

$$FM_{\text{wave}} \rightarrow f_c$$

$$PM_{\text{wave}} \rightarrow \phi$$

$$S(t) = A_c \cos(2\pi f_c t + \phi_c)$$

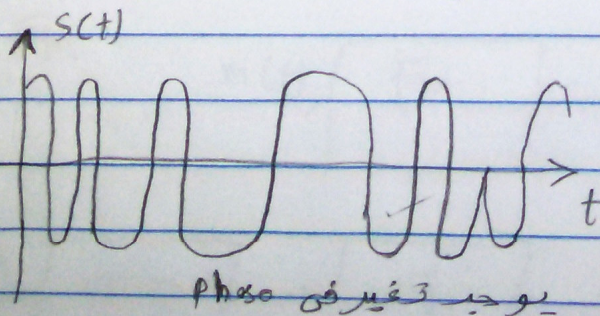
$\underbrace{2\pi f_c t}_{\text{un modulated Frequency}}$
 $\underbrace{\phi_c}_{\text{un modulated Angle}}$

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

$$PM_{\text{wave}} \rightarrow \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$k_p \rightarrow$ phase sensitivity

$$S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$



$$f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi \int_0^t f_i(t) dt$$

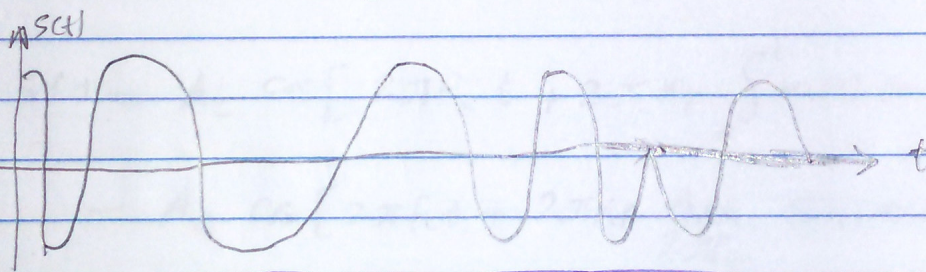
$$FM \rightarrow f_i(t) = f_c + K_f \cdot m(t)$$

$K_f \rightarrow$ frequency sensitivity

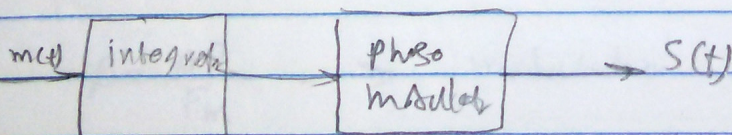
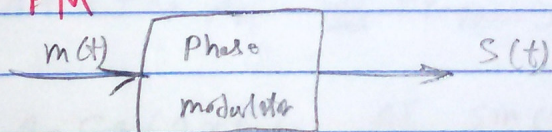
$$\phi_i(t) = 2\pi \int_0^t [f_c + K_f m(t)] dt$$

$$\phi_i(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt$$

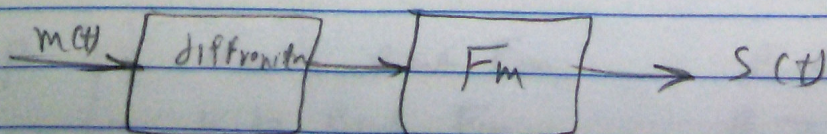
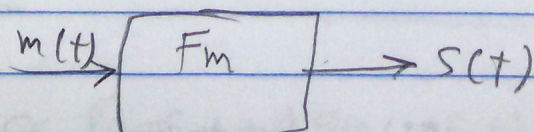
$$S(t) = A_c \cos [2\pi f_c t + 2\pi K_f \int_0^t m(t) dt]$$



FM from PM



PM from FM



FM and PM \rightarrow better than \rightarrow AM

نفسه فيها لا تتأثر بـ noise
 لا Amplitude ثابتة

وعليه هو تسمى الدائرة والتكلفة العالية

FM Type:

- ① single tone Fm
- ② multi tone Fm

* Single tone;

$$m(t) = A_m \cos(2\pi F_m t)$$

$$S(t) = A_c \cos \left[2\pi F_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$= A_c \cos \left[2\pi F_c t + 2\pi K_f \frac{A_m}{2\pi F_m} \sin(2\pi F_m t) \right]$$

$$\Delta F = \pm K_f A_m \quad \text{Frequency deviation}$$

$$S(t) = A_c \cos \left(2\pi f_c t + \frac{\Delta F}{F_m} \sin(2\pi F_m t) \right)$$

$$\beta = \frac{\Delta F}{F_m} \rightarrow \text{modulation index}$$

$$F_{\text{max}} = F_c \pm K_f A_m = F_c \pm \Delta F$$

$$S(t) = A_c \cos [2\pi f_c t + \beta \sin(2\pi F_m t)]$$

$\beta \rightarrow$ Narrow Band Fm $\beta < 1$
 $\beta \rightarrow$ Wide Band Fm $\beta > 1$

Narrow Band FM wave

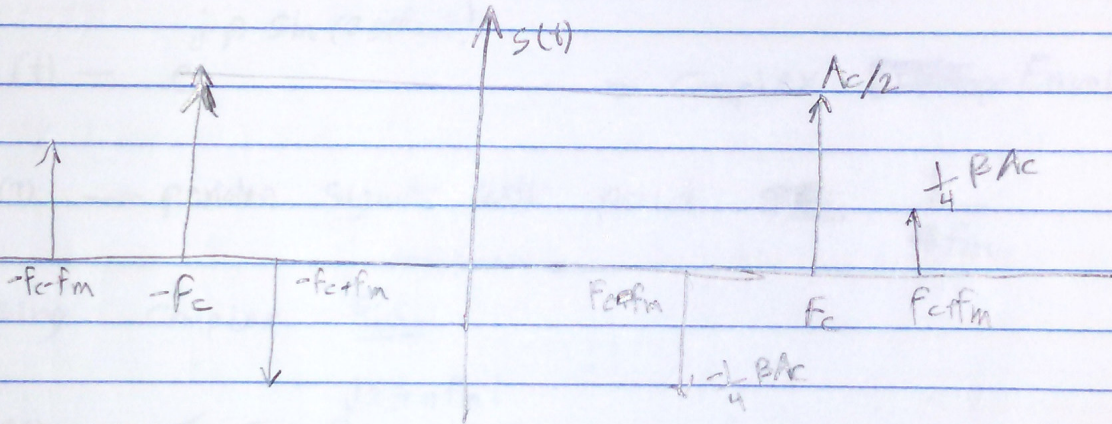
β is very small

$$S(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)]$$

$$- A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

$$\approx \beta \sin(2\pi f_m t)$$

$$S(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t)$$



$$S(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c [\cos(2\pi(f_c + f_m)t) - \cos(2\pi(f_c - f_m)t)]$$

$$B.W = 2f_m$$

* Wide Band Frequency Modulator

$$\beta \gg 1$$

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

$$= A_c \operatorname{Re} \left\{ e^{j2\pi f_c t + \beta \sin(2\pi f_m t)} \right\}$$

$$= A_c \operatorname{Re} \left\{ e^{j2\pi f_c t} \cdot e^{j\beta \sin(2\pi f_m t)} \right\}$$

$$\hat{S}(t) = e^{j\beta \sin(2\pi f_m t)}$$

→ Complex Envelope

$\hat{S}(t)$ → periodic signal with period $\frac{1}{f_m}$

Using Complex F.S.

$$\hat{S}(t) = \sum C_n e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} \hat{S}(t) \cdot e^{-j2\pi n f_m t} dt$$

$$= \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} dt$$

$$= \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

$$X = 2\pi f_m t$$

$$dX = 2\pi f_m dt$$

$$C_n = \frac{f_m}{2\pi f_m} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx$$

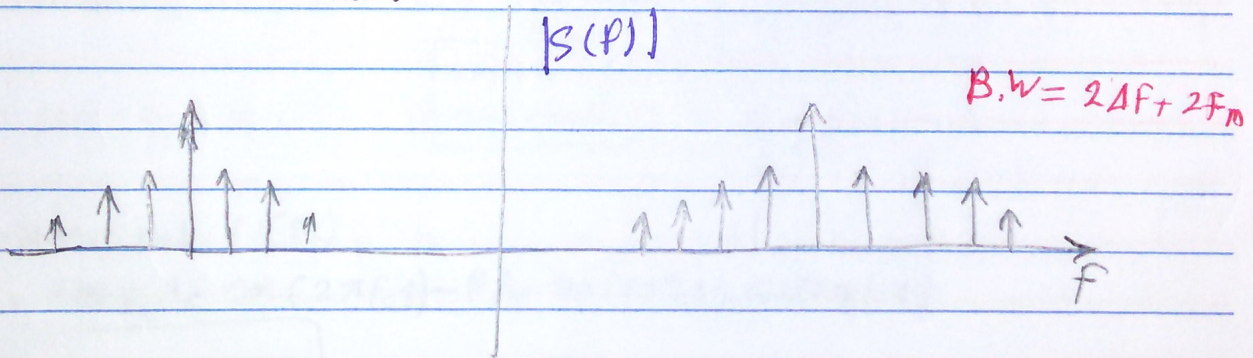
$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx \quad \text{Bessel Function}$$

$$C_n = J_n(\beta)$$

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t}$$

$$S(t) = A_c \operatorname{Re} \left\{ e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t} \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t)$$



$J_n(\beta)$ properties

$$n=0 \quad J_0(\beta) = 1$$

$$n = \text{even} \quad J_n(\beta) = J_{-n}(\beta)$$

$$n = \text{odd} \quad J_n(\beta) = -J_{-n}(\beta)$$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

* Generation of FM waves:

Transmission bandwidth of FM wave,

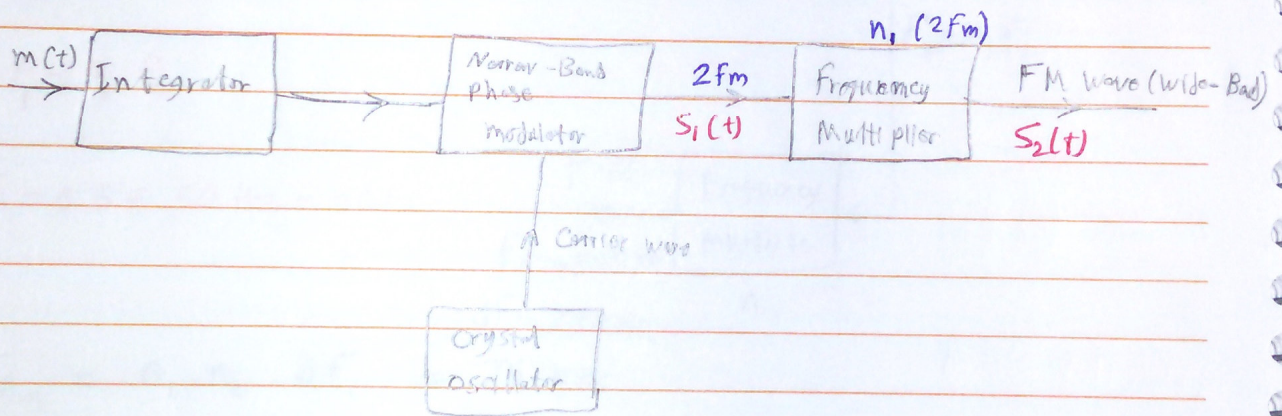
Carson's rule (wide Band FM)

$$B.W. \approx 2\Delta F + 2f_m \approx 2n_{max}f_m$$

$$|J_n(\beta)| > 0.01$$

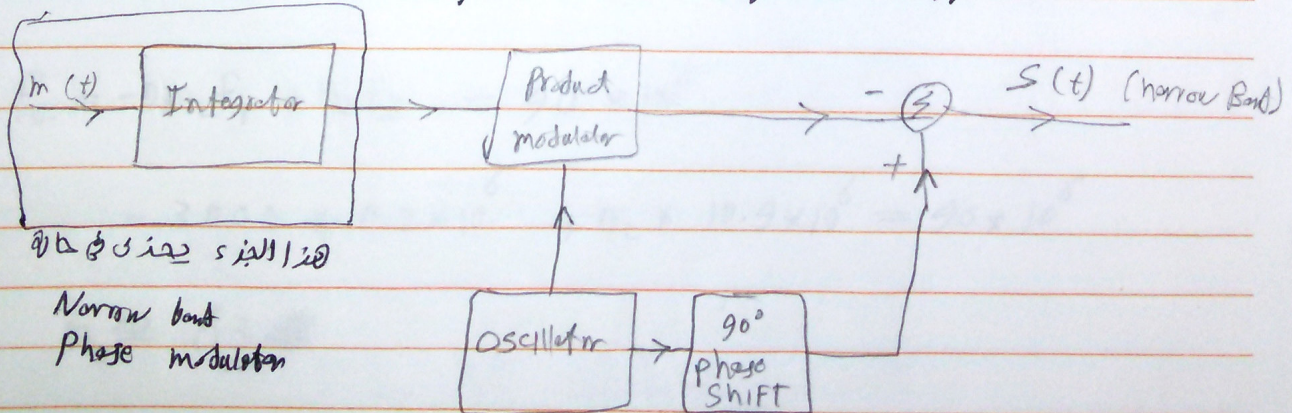
$$\Delta F = \frac{K_f A_m}{f_m} \approx \frac{2\pi K_f A_m}{f_m} f_m$$

* Indirect ~~method~~ FM generator:



Narrow Band (FM)

$$S(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

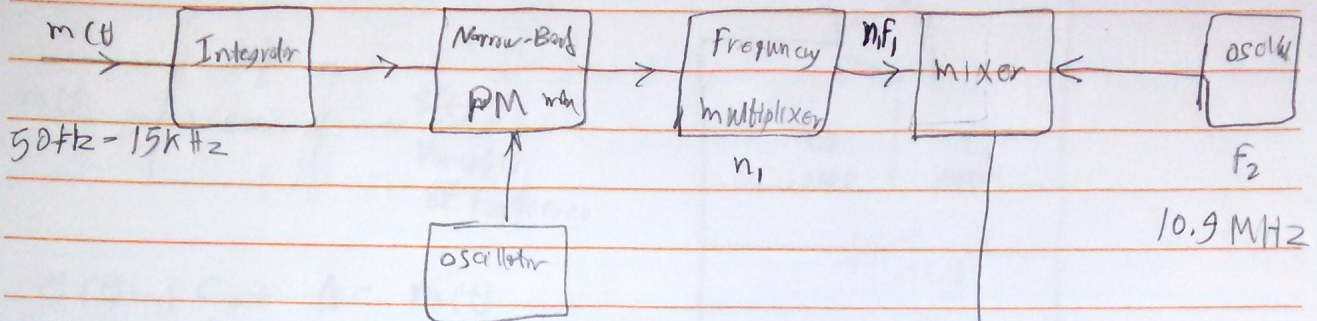


$$S_1(t) = A_1 \cos[2\pi f_c t + 2\pi K_f \int m(t) dt] \quad \text{if } m(t) = A_m \cos(2\pi f_m t)$$

$$S_1(t) = A_1 \cos[2\pi f_c t + \beta_1 \sin(2\pi f_m t)] \quad \beta_1 \ll 1$$

$$S_2(t) = A_1 \cos[2\pi f_c t + n_1 \beta_1 \sin(2\pi f_m t)] \quad n_1 \beta_1 \gg 1$$

Example : Find n_1, n_2



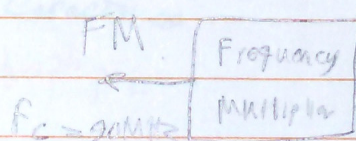
Condition (N.B)

$$\beta < 1$$

$$\beta_{\max} = 0.5$$

$$\Delta F_1 = \beta_1 \cdot F_m$$

$$\Delta F_1 = 0.5 \times 50 \text{ Hz} = 25 \text{ Hz}$$



$$\Delta F = 75 \text{ kHz}$$

$$\Delta F_{\text{final}} = n_1 \cdot n_2 \cdot \Delta F_1 = 75 \text{ kHz}$$

$$n_1, n_2 = 3000$$

$$F_c = -n_1 n_2 F_1 + n_2 F_2 = 90 \times 10^6$$

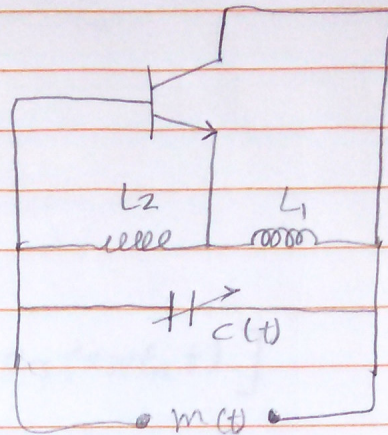
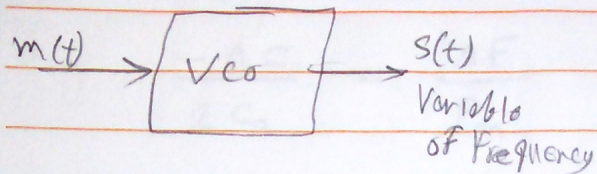
$$-3000 \times 0.2 \times 10^6 + n_2 \times 10.9 \times 10^6 = 90 \times 10^6$$

$$n_2 \approx 63$$

$$n_1 \approx 48$$

* Direct method, Direct FM generator

* Voltage Controlled Oscillator



$$C(t) = C_0 + \Delta C m(t)$$

\downarrow
 Capacitance in the absence of modulation
 \downarrow
 base band signal

Hartley oscillator
VCO

$\Delta C \rightarrow$ max change of capacitance

max change of capacitance

P-n Junction with reverse bias

~~Varactor~~ \rightarrow Varactor, or Vari Cap

$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}} = \frac{1}{2\pi \sqrt{(L_1 + L_2) [C_0 + \Delta C m(t)]}}$$

$$\text{if } m(t) = \cos(2\pi f_m t)$$

$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$

$$f_i(t) = f_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) \right]^{\frac{1}{2}}$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$

$$F_1(t) = F_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) + \dots \right]$$

$$\boxed{\Delta C < C_0}$$

$$\frac{-\Delta C}{2C_0} = \frac{\Delta F}{F_m}$$

$$\therefore F_1(t) = F_0 \left[1 + \frac{\Delta F}{F_m} \cos(2\pi f_m t) \right]$$

Instantaneous Frequency of FM wave

Disadvantage \rightarrow * Un stable Frequency

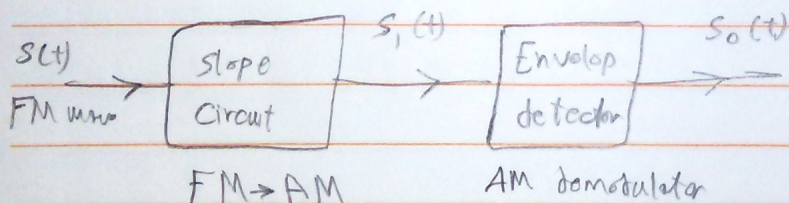
* Variable Capacitance is not Linear

* Detection of FM wave:

① Slope detector

② PLL \rightarrow Phase Locked - Loop

① Slope detector



Slope circuit \rightarrow Differentiator

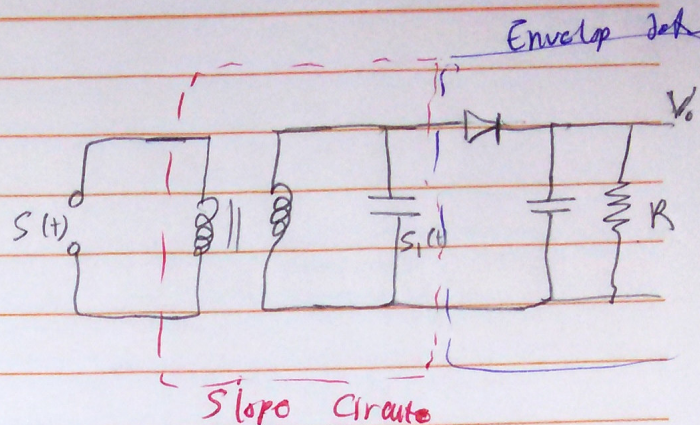
$$S(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$S_1(t) = -A_c [2\pi f_c + 2\pi K_f m(t)] \sin \left[2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$S_0(t) = -A_c [2\pi f_c + 2\pi K_f m(t)]$$

$$S_0(t) \propto m(t)$$

Slope Circuits



② Phase-Locked-Loop:

